Tanta University — Fourth Year Students
Faculty of Engineering — 2009 / 2010
Computers and Control Department — First Semester

Neural Networks

This set of problems is intended to acquaint the student with neurons employing binary / bipolar threshold signals and how simple neural networks can be used to implement logic (boolean) functions.

Problem 1

Consider the neural network illustrated in Fig. 1. The inputs are $x_1 = -1$, $x_2 = -2$, and $x_3 = 1$. The output neuron produces a binary threshold signal s (that is, $s = 1$ for $y > 0$ and $s = 0$ for $y < 0$, where y is the activation). Determine s for weight values $w_{14} = -1$, $w_{24} = 1.5$, $w_{34} = 2$, and $w_{04} = -0.5$.

Problem 2

Repeat Prob. 1 for weight values $w_{14} = 0.5$, $w_{24} = -2$, $w_{34} = -1.5$, and $w_{04} = -0.8$.

Problem 3

In Prob. 1, let $w_{14} = w_{24} = w_{34} = 0.5$. Find the value of the bias weight w_{04} such that the activation y is zero.

Problem 4

Repeat Prob. 1 when the output neuron produces a bipolar threshold signal s (that is, $s = 1$ for $y > 0$ and $s = -1$ for $y < 0$).

Problem 5

Repeat Prob. 1 when the output neuron produces a bipolar threshold signal and the weights take on the values given in Prob. 2.

Problem 6

Consider the neural network illustrated in Fig. 2. The output neuron produces a binary threshold signal. Show that this network implements a logic NOT function (inverter) when $w_{12} = -2$ and $w_{02} = 1$.

Give other values for w_{12} and w_{02} that likewise satisfy the NOT function implementation.

Problem 7

Consider the neural network illustrated in Fig. 3. The output neuron produces a binary threshold signal. Show that this network implements a logic AND function when $w_{13} = 2$, $w_{23} = 2$, and $w_{03} = -3.5$.

Give other values for w_{13}, w_{23}, and w_{03} that likewise satisfy the AND function implementation.

Problem 8

Show that the neural network of Fig. 3 implements a logic OR function when $w_{13} = 2$, $w_{23} = 2$, and $w_{03} = -1.5$.

Give other values for w_{13}, w_{23}, and w_{03} that likewise satisfy the OR function implementation.

Problem 9

Find values for w_{13}, w_{23}, and w_{03} such that the neural network of Fig. 3 implements a logic NAND function. (You may make use of the solution of Prob. 7).

Problem 10

Find values for w_{13}, w_{23}, and w_{03} such that the neural network of Fig. 3 implements a logic NOR function. (You may make use of the solution of Prob. 8).

Problem 11

In the neural network of Fig. 3, let $w_{13} = -1$, $w_{23} = 2.5$, and $w_{03} = -1.6$. Show that the network now implements a logic function $x_1' x_2$, where the prime denotes complementation.

Problem 12

In the neural network of Fig. 3, let $w_{13} = 2.5$, $w_{23} = -1$, and $w_{03} = -1.8$. What logic function is now implemented by the network?

Problem 13

Consider the neural network illustrated in Fig. 4. All neurons in the hidden and output layers employ binary threshold signals. Show that this network implements a logic XOR function when

$w_{13} = 2$ $\quad w_{14} = -1$ $\quad w_{23} = -1$

$w_{24} = 2$ $\quad w_{35} = 2$ $\quad w_{45} = 2$

$w_{03} = -1.5$ $\quad w_{04} = -1.5$ $\quad w_{05} = -1.5$

Problem 14

Show that the neural network of Fig. 4 implements a logic XNOR function when

$w_{13} = -2$ $\quad w_{14} = 1$ $\quad w_{23} = 1$

$w_{24} = -2$ $\quad w_{35} = 1$ $\quad w_{45} = 1$

$w_{03} = 1.5$ $\quad w_{04} = 1.5$ $\quad w_{05} = -1.5$

Problem 15

Convince yourself that a neural network of the form of Fig. 3 cannot implement either an XOR or an XNOR function.

Problem 16

Prove that a logic XOR operation can be expressed as

$$x_1 \oplus x_2 = (x_1 x_2)' (x_1 + x_2)$$

that is, a NAND operation ANDed with an OR operation.

Use this relation to construct a three-layer neural network that implements the XOR function.

Problem 17

Prove that a logic XNOR operation can be expressed as

$$x_1 \odot x_2 = (x_1 + x_2)' + x_1 x_2$$

that is, a NOR operation ORed with an AND operation.

Use this relation to construct a three-layer neural network that implements the XNOR function.

Problem 18

Consider the neural network illustrated in Fig. 5. The two hidden neurons have bipolar sigmoidal functions and the output neuron has a bipolar threshold function. Determine the value of the response signal s when the network inputs are $x_1 = 1.3$, $x_2 = -0.9$, and $x_3 = -0.4$.

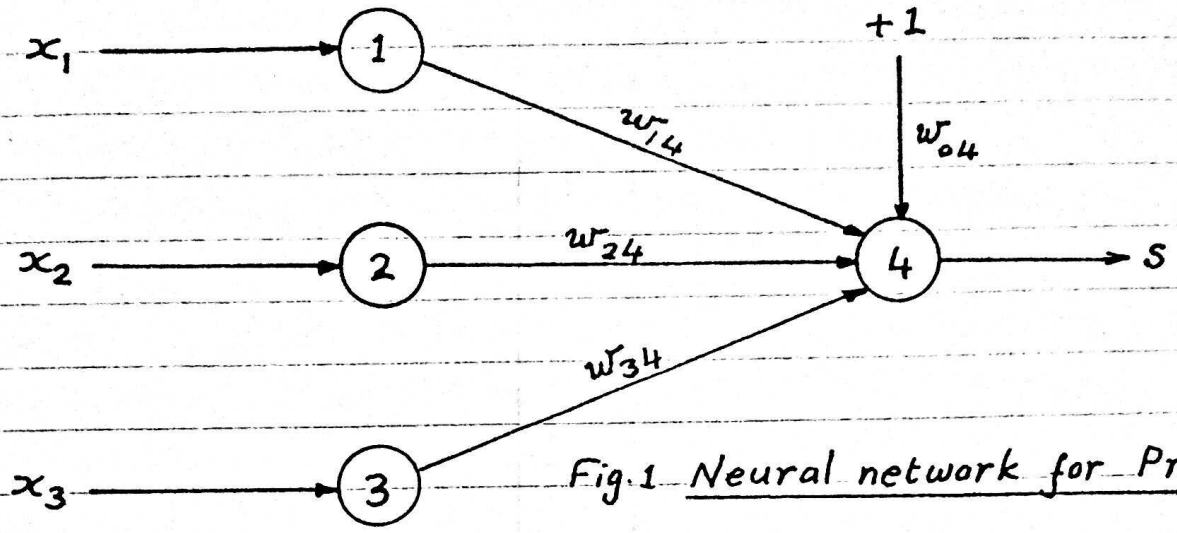

Fig. 1 Neural network for Prob. 1

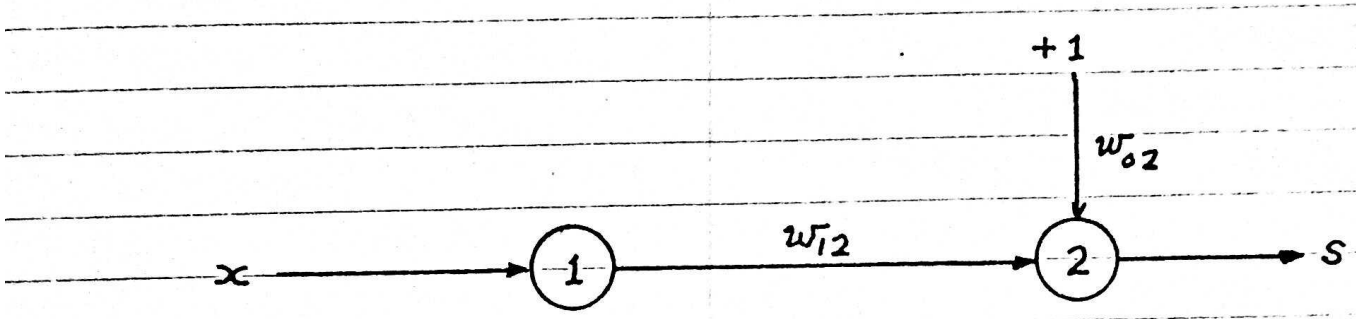

Fig. 2 Neural network for Prob. 6

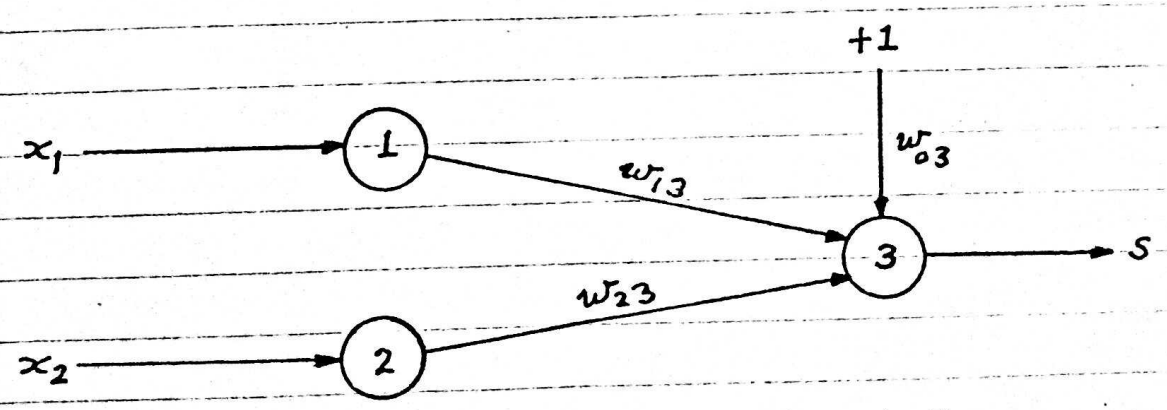

Fig. 3 Neural network for Prob. 7

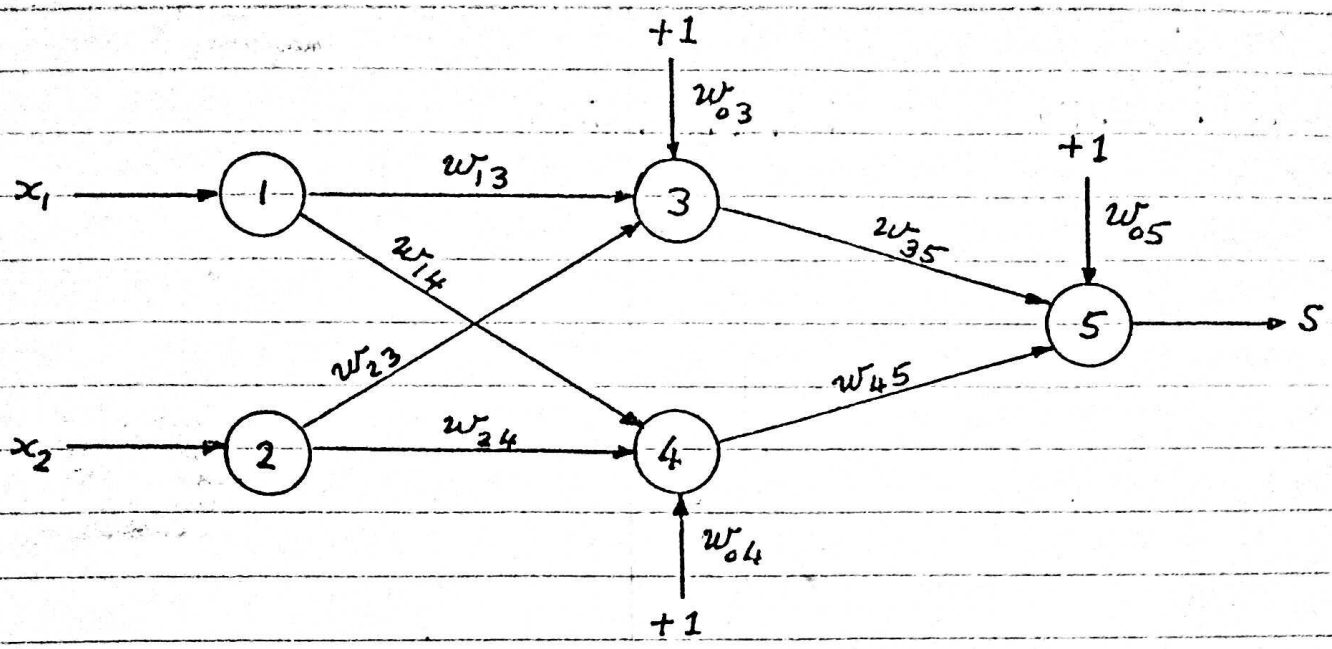

Fig. 4 Neural network for Prob. 13

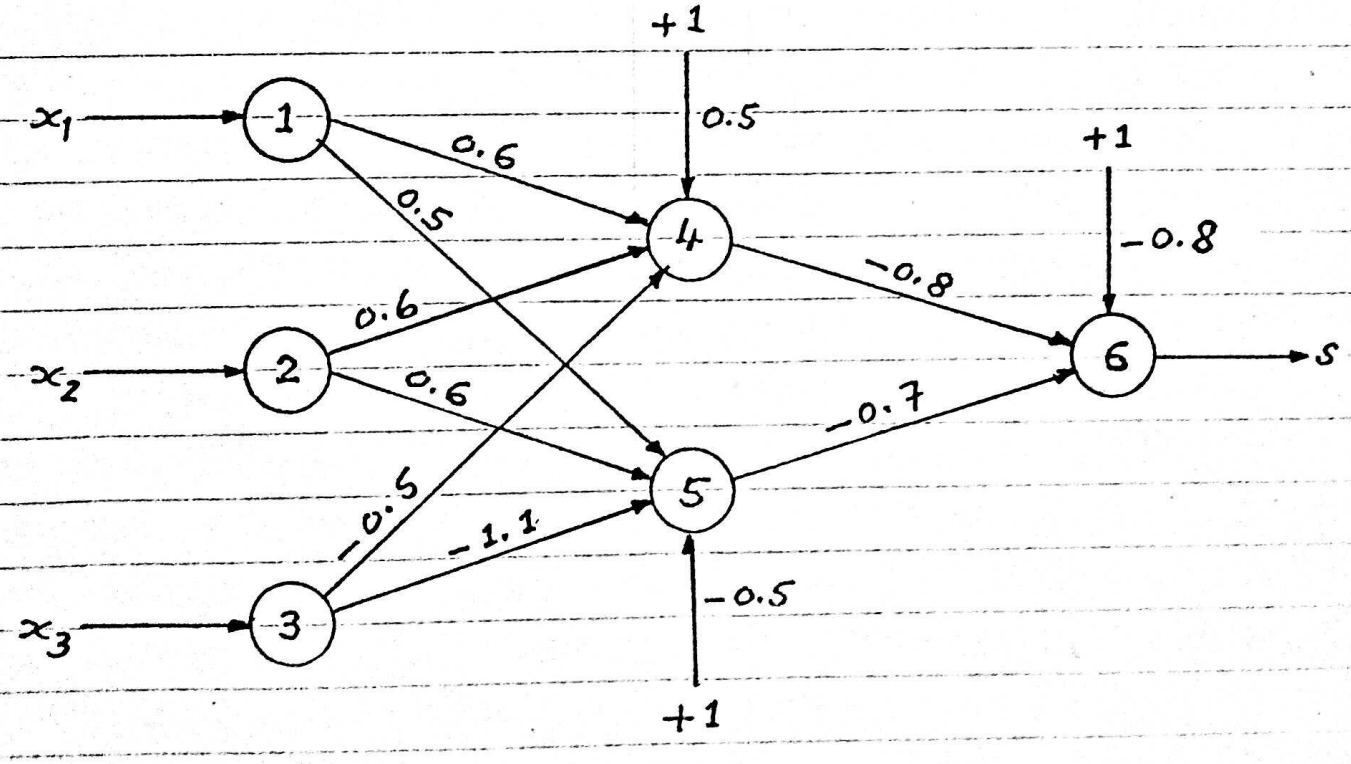

Fig. 5 Neural network for Prob. 18